Alternatives to General Relativity (GR).

There have been hundreds of different attempts at constructing an ideal theory of gravity. It's not possible to cover all of them. However, these attempts do tend to fall into four categories: those that are straightforward alternatives to General Relativity (GR) such as the Cartan, Brans-Dicke and Rosen bimetric theories, those that attempt to construct a quantized gravity theory such as loop quantum gravity, those that attempt to unify gravity and other forces such as Kaluza-Klein, and those that attempt to do several at once such as M-theory.

This article deals only with straightforward alternatives to GR. For quantized gravity theory see article [?]. For the unification of gravity and other forces see article [General Unified Theories]. For those theories that attempt to do several at once see article [Theory of Everything].

General Relativity is now more than 90 years old, but while one alternative theory of gravity after another has failed to agree with ever more accurate observations, Einstein (1916) theory of General Relativity has stood firm. Occasional discrepancies between GR and observation are reported, these discrepancies have all so far evaporated as more accurate observations have been done.

1. Motivations

Motivations for developing new theories of gravity have changed over the years, the first was to explain planetary orbits (Newton) and more complicated orbits (eg. Lagrange). Then came unsuccessful attempts to combine gravity and either wave or corpuscular theories of gravity. The whole landscape of physics was changed with the discovery of Lorentz transformations, and this led to attempts to reconcile it with gravity. At the same time, experimental physicists started testing the foundations of gravity and relativity - Lorentz invariance, the gravitational deflection of light, the Eötvös experiment. These considerations led to and past the development of General Relativity.

After that, motivations differ. Two major concerns were the development of quantum theory and the discovery of the strong and weak nuclear forces. Attempts to quantize and unify gravity are outside the scope of this article, and so far none has been completely successful.

After GR, attempts were made to either improve on theories developed before GR, or to improve GR itself. Many different strategies were attempted, for example the addition of spin to GR, combining a GR-like metric with a space-time that is static with respect to the expansion of the universe, getting extra freedom by adding another parameter. At least one theory was motivated by the desire to develop an alternative to GR that is completely free from singularities.

Experimental tests improved along with the theories. Many of the different strategies that were developed soon after GR were abandoned, and there was a push to develop more general forms of the theories that survived, so that a theory would be ready the moment any test showed a disagreement with GR.

By the 1980s, the increasing accuracy of experimental tests had all led to confirmation of GR, no competitors were left except for those that included GR as a special case, and they can be rejected on the grounds of Occam's Razor until an experimental discrepancy shows up. Further, shortly after that, theorists switched to string theory which was starting to look promising. In the mid 1980s a few experiments were suggesting that gravity was being modified by the addition of a fifth force (or, in one case, of a fifth, sixth and seventh force) acting on the scale of metres. Subsequent experiments eliminated these.

Motivations for the more recent alternative theories are almost all cosmological, associated with or replacing such constructs as "inflation", "dark matter" and "dark energy". The basic idea is that gravity agrees with GR at the present epoch but may have been quite different in the early universe. Investigation of the Pioneer anomaly has caused renewed public interest in alternatives to General Relativity, but the Pioneer anomaly is too strong to be explained by any such theory of gravity.

2. Early theories, 1686 to 1916

For more details see Pais (1982). See also article [History of General Relativity].

Early theories of gravity, those before General Relativity, include those of Newton (1686), Einstein (1912a & b), Einstein and Grossmann (1913), Nordstrom (1912, 1913) and Einstein and Fokker (1914).

In Newton's (1686) theory (rewritten using more modern mathematics) the density of mass ρ generates a scalar field ϕ by:

$$\nabla^2 \phi \equiv \frac{\partial^2 \phi}{\partial x^j \partial x^j} = 4\pi\rho$$

and the scalar field governs the motion of a freely falling particle by:

$$\frac{d^2 x^j}{d t^2} + \frac{\partial \phi}{\partial x^j} = 0$$

where the scalar field is,

$$\phi = GM/r$$

Some theories of gravity between those of Newton and Einstein include those of Lagrange (applying the variational principle), Le Sage (a corpuscular model), Tomasina (a wave model) and Lorentz (a wave model). Poincaré (1908) compares these and concludes that Lagrange's version is correct*. The other models predict very large super-luminal velocities for the speed of gravity and this in turn would lead to extremely rapid heating of the Earth, which doesn't happen.
* [Wikipedia Le Sage]

The theory of Newton (1686), and Lagrange's improvement on the calculation, completely fails to take into account relativistic effects of course, and so can be rejected as a viable theory of gravity. Even so, Newton's theory is thought to be exactly correct in the limit of weak gravitational fields and low speeds and all other theories of gravity need to reproduce Newton's theory in the appropriate limits.

Einstein's two part publication in 1912 is really only important for historical reasons. By then he knew of the the gravitational redshift and the deflection of light. He had realized that Lorentz transformations are not generally applicable, but retained them. The theory states that the speed of light is constant in free space but varies in the presence of matter. The theory was only expected to hold when the source of the gravitational field is stationary. It includes the principle of least action:

$$\delta \int ds = 0$$

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu$$

where η is the Minkowski metric, and there is a summation from 1 to 4 over indices μ and ν.

Einstein and Grossmann (1913) includes Riemannian geometry and tensor calculus.

$$\delta \int ds = 0$$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

The equations of electrodynamics exactly match those of GR. The equation

$$T_{\mu\nu}=\kappa\rho_0\frac{dx^\mu}{ds}\frac{dx^\nu}{ds}$$

is not in GR. It expresses the stress energy tensor as a function of the matter density.

While this was going on, Abraham was developing an alternative model of gravity in which the speed of light depends on the gravitational field strength and so is variable almost everywhere. Abraham's 1914 review of gravitation models is said to be excellent, but his own model was poor.

The first approach of Nordström (1912) was to retain the Minkowski metric and a constant value of *c* but to let mass depend on the gravitational field strength φ. Allowing this field strength to satisfy

$$\Box\phi=\rho$$

where ρ is rest mass energy and $\Box$ is the d'Alembertian,

$$m=m_0\exp(\phi/c^2)$$

and

$$-\frac{\partial\phi}{\partial x^\mu}=\dot{u}_\mu+\frac{u_\mu}{c^2}\dot{\phi}$$

where *u* is the four-velocity and the dot is a differential with respect to time.

The second approach of Nordström (1913) is remembered as the the first logically consistent relativistic field theory of gravitation ever formulated. From (note, notation of Pais (1982) not Nordström):

$$\delta\int\psi\,ds=0$$

$$ds^2=\eta_{\mu\nu}dx^\mu dx^\nu$$

where ψ is a scalar field,

$$\frac{\partial T^{\mu\nu}}{\partial x^\nu}=T\frac{1}{\psi}\frac{\partial\psi}{\partial x_\mu}$$

This theory is Lorentz invariant, satisfies the conservation laws, correctly reduces to the Newtonian limit and satisfies the weak equivalence principle.

Einstein and Fokker (1914) is Einstein's first treatment of gravitation in which general covariance is strictly obeyed. Writing:

$$\delta\int ds=0$$

$$ds^2=-g_{\mu\nu}dx^\mu dx^\nu$$

$$g_{\mu\nu}=\psi^2\eta_{\mu\nu}$$

they relate Einstein-Grossmann (1913) to Nordström (1913). They also state:

$$T\propto R$$

That is, the trace of the stress energy tensor is proportional to the curvature of space.

Einstein (1916, 1917) is what we now know of as General Relativity. Discarding the Minkowski metric entirely, Einstein gets:

$$\delta\int ds=0$$

$$ds^2=-g_{\mu\nu}dx^\mu dx^\nu$$

$$R_{\mu\nu}=8\pi G(T_{\mu\nu}-g_{\mu\nu}T/2)$$

which can also be written

$$T^{\mu\nu}=\frac{1}{8\pi G}(R^{\mu\nu}-g^{\mu\nu}R/2)$$

Five days before Einstein presented the last equation above, Hilbert had submitted a paper containing an almost identical equation. See [controversy]. Hilbert was the first to correctly state the Einstein-Hilbert action for GR, which is:

$$S=\frac{1}{16\pi G}\int R\sqrt{-g}\,d^4x+S_m$$

where G is Newton's gravitational constant, $R=R_{\mu\mu}$ is the curvature of space, $g=g_{\mu\mu}$ and S_m is the action due to mass.

GR is a tensor theory, the equations all contain tensors. Nordström's theories, on the other hand, are scalar theories because the gravitational field is a scalar. Later in this article you will see scalar-tensor theories that contain a scalar field in addition to the tensors of GR, and other variants containing vector fields as well have been developed recently.

3. Theories from 1917 to the 1980s

This section includes alternatives to GR published after GR but before the observations of galaxy rotation that led to the hypothesis of "dark matter".

Those considered here include (see Will (1981)*, Turyshev (2006)*):
* Some further references can be found in Ni (1972), they don't add much more to the overall picture.
* - Although an important source for this article, Turyshev's presentation contains many errors of fact.
Whitehead (1922), Cartan (1922, 1923), Fierz & Pauli (1939), Birkhov (1943), Milne (1948), Thiry (1948), Papapetrou (1954a, 1954b), Littlewood (1953), Jordan (1955), Bergman (1956), Belinfante & Swihart (1957), Yimatz (1958, 1973), Brans & Dicke (1961), Whitrow & Morduch (1960, 1965), Kustaanheimo (1966) , Kustaanheimo & Nuotio (1967), Deser & Laurent (1968), Page & Tupper (1968), Bergmann (1968), Bollini-Giambini-Tiomno (1970), Nordtveldt (1970), Wagoner (1970), Rosen (1971, 1975, 1975), Ni (1972, 1973), Will & Nordtveldt (1972), Hellings & Nordtveldt (1973), Lightman & Lee (1973), Lee, Lightman & Ni (1974), Beckenstein (1977), Barker (1978), Rastall (1979)

In this article, these theories are presented without a cosmological constant or added scalar or vector potential unless specifically noted, for the simple reason that the need for one or both of these was not recognised before the supernova observations by Perlmutter.

3.1. Classification of Theories

Theories of gravity can be classified, loosely, into several categories. Most of the theories described here have:
i) an 'action' (also known as a variational principle)
ii) a Lagrangian density
iii) a metric

If a theory has a Lagrangian density, say L, then the action S is the integral of that, for example
$$S\propto\int d^4x\,R\sqrt{-g}\,L$$
where R is the curvature of space. In this equation it is usual, though not essential, to have $g=-1$

Almost every theory described in this article has an action. It is the only known way to guarantee that the necessary conservation laws of energy, momentum and angular momentum are incorporated automatically; although it is easy to construct an action where those conservation laws are violated. The original 1983 version of MOND did not have an action.

A few theories have an action but not a Lagrangian. A good example is Whitehead (1922), the action there is termed non-local.

A theory of gravity is a "metric theory" if and only if it can be given a mathematical representation in which two conditions hold:
Condition 1. There exists a metric $g_{\mu\nu}$ of signature -2, which governs proper-length and proper-time measurements in the usual manner of special and general relativity:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

where there is a summation over indices μ and ν.
Condition 2. Stressed matter and fields being acted upon by gravity respond in accordance with the equation:

$$\nabla \cdot T = 0$$

where T is the stress-energy tensor for all matter and non-gravitational fields, and where ∇ is the covariant derivative with respect to the metric.

Any theory of gravity in which $g_{\mu\nu} \neq g_{\nu\mu}$ is always true is not a metric theory, but any metric theory can perfectly well be given a mathematical description that violates conditions i and ii.

Metric theories include (from simplest to most complex):
a) Scalar field theories (includes Conformally flat theories & Stratified theories with conformally flat space slices)
Nordström, Einstein-Fokker, Whitrow-Morduch, Littlewood, Bergman, Page-Tupper, Einstein (1912), Whitrow-Morduch, Rosen (1971), Papapetrou, Ni, Yilmaz, [Coleman], Lee-Lightman-Ni
b) Bimetric theories
Rosen (1975), Rastall, Lightman-Lee
c) Quasilinear theories (includes Linear fixed gauge)
Whitehead, Deser-Laurent, Bollini-Giambini-Tiomno
d) Tensor-theories
Einstein's GR
e) Scalar-tensor theories
Thiry, Jordan, Brans-Dicke, Bergmann, Wagoner, Nordtvedt, Beckenstein
f) Vector-tensor theories
Will-Nordtvedt, Hellings-Nordvedt
g) Other metric theories
(see section Modern Theories below)

h) Non-metric theories
Cartan, Belinfante-Swihart

A word here about Mach's principle is appropriate because a few of these theories rely on Mach's principle eg. Whitehead (1922), and many mention it in passing eg. Einstein-Grossmann (1913), Brans-Dicke (1961). Mach's principle can be thought of a half-way-house between Newton and Einstein. It goes this way*:
* this isn't exactly the way Mach originally stated it, but is the way it has been interpreted.

Newton: Absolute space and time
Mach: The reference frame comes from the distribution of matter in the universe
Einstein: There is no reference frame

So far, all the experimental evidence points to Mach's principle being wrong, but it has not entirely been ruled out.

3.2. Scalar Field Theories

The scalar field theories of Nordström (1912, 1913) have already been discussed. Those of Littlewood (1953), Bergman (1956), Yilmaz (1958), Whitrow and Morduch (1960, 1965) and Page and Tupper (1968) follow the general formula give by Page and Tupper.

According to Page and Tupper (1968), who discuss all these except Nordström (1913), the general scalar field theory comes from the principle of least action:

$$\delta \int f(\phi/c^2)ds$$

where the scalar field is,

$$\phi = GM/r$$

and c may or may not depend on φ.

In Nordström (1912),

$$f(\phi/c^2)=\exp(-\phi/c^2) \quad ; \quad c=c_\infty$$

In Littlewood (1953) and Bergmann (1956),

$$f(\phi/c^2)=\exp(-\phi/c^2-(\phi/c^2)^2/2) \quad ; \quad c=c_\infty$$

In Whitrow and Morduch (1960),

$$f(\phi/c^2)=1 \quad ; \quad c^2=c_\infty^2-2\phi$$

In Whitrow and Morduch (1965),

$$f(\phi/c^2)=\exp(-\phi/c^2) \quad ; \quad c^2=c_\infty^2-2\phi$$

In Page and Tupper (1968),

$$f(\phi/c^2)=\phi/c^2+\alpha(\phi/c^2)^2 \quad ; \quad c_\infty^2/c^2=1+4(\phi/c_\infty^2)+(15+2\alpha)(\phi/c_\infty^2)^2$$

Page and Tupper (1968) matches Yilmatz (1958) to second order when $\alpha=-7/2$.

The gravitational deflection of light has to be zero when *c* is constant. Given that variable *c* and zero deflection of light are both in conflict with experiment, the prospect for a successful scalar theory of gravity looks very unlikely. Further, if the parameters of a scalar theory are adjusted so that the deflection of light is correct then the gravitational redshift is likely to be wrong.

In Ni (1972), a pre-existing special relativity space-time and universal time coordinate acts with matter and non-gravitational fields to generate a scalar field. This scalar field acts together with all the rest to generate the metric. The scalar field is the "rest frame of the universe", not a silly idea because the cosmic microwave background and the recession of galaxies individually and together define a preferred Lorentzian frame.

The action is:

$$S=\frac{1}{16\pi}\int d^4x\sqrt{g}\,L_\phi+S_m$$

$$L_\phi=\phi R-2g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \quad \text{where} \quad \partial_\mu\equiv\frac{\partial}{\partial x^\mu}$$

S_m is the matter action.

$$\Box\phi=4\pi T^{\mu\nu}[\eta_{\mu\nu}e^{-2\phi}+(e^{2\phi}+e^{-2\phi})\partial_\mu t\,\partial_\nu t]$$

t is the universal time coordinate.

This theory is self-consistent and complete. But the motion of the solar system through the universe leads to serious disagreement with experiment.

In Papapetrou (1954a) the gravitational part of the Lagrangian is:

$$L_\phi = e^{\phi}((1/2)e^{-\phi}\partial_\alpha\phi\partial_\alpha\phi + (3/2)e^{\phi}\partial_0\phi\partial_0\phi)$$

In Papapetrou (1954b) there is a second scalar field χ. The gravitational part of the Lagrangian is now:

$$L_\phi = e^{(3\phi+\chi)/2}(-(1/2)e^{-\phi}\partial_\alpha\phi\partial_\alpha\phi - e^{-\phi}\partial_\alpha\phi\partial_\alpha\chi + (3/2)e^{-\chi}\partial_0\phi\partial_0\phi)$$

3.3. Bimetric theories

Bimetric theories contain both the normal tensor metric and the Minkowski metric η (or a metric of constant curvature), and may contain other scalar or vector fields.

Rosen (1971 [?], 1973, 1975) Bimetric Theory
The Action is:

$$S = \frac{1}{64\pi G}\int d^4x\sqrt{-\eta}\,\eta^{\mu\nu}g^{\alpha\beta}g^{\gamma\delta}(g_{\alpha\gamma|\mu}g_{\alpha\delta|\nu} - (1/2)g_{\alpha\beta|\mu}g_{\gamma\delta|\nu}) + S_m$$

where the vertical line "|" denotes covariant derivative with respect to $\boldsymbol{\eta}$. The field equations may be written in the form:

$$\Box_\eta g_{\mu\nu} - g^{\alpha\beta}\eta^{\gamma\delta}g_{\mu\alpha|\gamma}g_{\nu\beta|\delta} = -16\pi G\sqrt{g/\eta}(T_{\mu\nu} - (1/2)g_{\mu\nu}T)$$

Rastall (1979)
The metric is an algebraic function of the Minkowski metric and a Vector field K_μ.*
* Will (1981) lists this as bimetric but I don't see why it isn't just a vector field theory.
The Action is:

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,F(N)K^{\mu;\nu}K_{\mu;\nu} + S_m$$

where

$F(N) = -N/(2+N)$ and $N = g^{\mu\nu}K_\mu K_\nu$

(see Will (1981) for the field equation for $T^{\mu\nu}$ and K_μ).

Lightman-Lee (1973)
This is a metric theory based on the non-metric theory of Belinfante and Swihart (1957a, 1957b).

3.4. Quasilinear Theories

In Whitehead (1922), the physical metric g is constructed algebraically from the Minkowski metric η and matter variables, so it doesn't even have a scalar field. The construction is:

$$g_{\mu\nu}(x^\alpha) = \eta_{\mu\nu} - 2\int_{\Sigma^-} y_\mu^- \frac{y_\nu^-}{(w^-)^3}[\sqrt{(g)}\rho u^\alpha d\Sigma_\alpha]^-$$

where the superscript (-) indicates quantities evaluated along the past η-light cone of the field point x^α and

$(y^\mu)^- = x^\mu - (x^\mu)^-$, $(y^\mu)^-(y_\mu)^- = 0$, $w^- = (y^\mu)^-(u_\mu)^-$, $(u_\mu) = dx^\mu/d\sigma$,
$\sigma^2 = \eta_{\mu\nu}dx^\mu dx^\nu$

Deser and Laurent (1968) and Bollini-Giambini-Tiomno (1970) are Linear Fixed Gauge (LFG) theories. Taking an approach from quantum field theory, combine a Minkowski spacetime with the gauge invariant action of a spin-two tensor field (i.e. graviton) $h_{\mu\nu}$ to set

$$g_{\mu\nu} \equiv \eta_{\mu\nu} + h_{\mu\nu}$$

The action is:

$$S=\frac{1}{16\pi G}\int d^4x\sqrt{-\eta}[2h^{\mu\nu}_{|\nu}h_{\mu\lambda}^{|\lambda}-2h^{\mu\nu}_{|\nu}h^{\lambda}_{\lambda|\mu}+h^{\nu}_{\nu|\mu}h^{\lambda|\mu}_{\lambda}-h^{\mu\nu|\lambda}h_{\mu\nu|\lambda}]+S_m$$

The Bianchi identity associated with this partial gauge invariance is wrong. LFG theories seek to remedy this by breaking the gauge invariance of the gravitational action through the introduction of auxiliary gravitational fields that couple to $h_{\mu\nu}$.

[Whitehead (1922) theory of gravity also predicts relativistic effects reliably but predicts a time-dependence for the ocean tides that is contradicted by everyday experience. [Expand, preferred reference frame, sidereal]]

3.5. Tensor Theories

There is only one theory that possesses a single gravitational field that is the metric. Of course this theory is General Relativity.

3.6. Scalar-Tensor Theories

These all contain at least one free parameter, as opposed to GR which has no free parameters.

Although not normally considered a Scalar-Tensor theory of gravity, the 5 by 5 metric of Kaluza-Klein reduces to a 4 by 4 metric and a single scalar. So if the 5th element is treated as a scalar gravitational field instead of an electromagnetic field then Kaluza-Klein can be considered the progenitor of Scalar-Tensor theories of gravity. This was recognised by Thiry (1948).

Scalar-Tensor theories include Thiry (1948), Jordan (1955), Brans and Dicke (1961), Bergman (1968), Nordtveldt (1970), Wagoner (1970), Bekenstein (1977) and Barker (1978).

The action S is based on the integral of the Lagrangian L_φ.

$$S=\frac{1}{16\pi}\int d^4x\sqrt{-g}\,L_\phi+S_m$$

$$L_\phi=\phi R-\frac{\omega(\phi)}{\phi}g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi+2\phi\lambda(\phi)$$

$$S_m=\int d^4x\sqrt{g}\,G_N L_m$$

$$T^{\mu\nu}\equiv\frac{2}{\sqrt{g}}\frac{\delta S_m}{\delta g_{\mu\nu}}$$

where $\omega(\phi)$ is a different dimensionless function for each different scalar-tensor theory. The function $\lambda(\phi)$ plays the same role as the cosmological constant in GR. The full version is retained in Bergman (1968) and Wagoner (1970).

Special cases are:

Nordtveldt (1970), $\lambda=0$ *
* I'm not worrying about this term until later in the article, it's discussed under modern theories.

Brans-Dicke (1961), ω = constant

Bekenstein (1977) Variable Mass Theory
Starting with parameters r and q, found from a cosmological solution,
$\phi=[1-qf(\phi)]f(\phi)^{-r}$ determines function f then

$$\omega(\phi)=-3/2-(1/4)f(\phi)[(1-6q)qf(\phi)-1][r+(1-r)qf(\phi)]^{-2}$$

Barker (1978) Constant G Theory

$$\omega(\phi)=(4-3\phi)/(2\phi-2)$$

It is constant in the Brans-Dicke (1961) theory. G_N is a dimensionless normalization constant that fixes the present-day value of G. An arbitrary potential can be added for the scalar.

Adjustment of $\omega(\phi)$ allows Scalar Tensor Theories to tend to GR in the limit of $\omega\rightarrow\infty$ in the current epoch. However, there could be significant differences from GR in the early universe.

So long as GR is confirmed by experiment, general Scalar-Tensor theories (including Brans-Dicke) can never be ruled out entirely, but as experiments continue to confirm GR more precisely and the parameters have to be fine-tuned so that the predictions more closely match those of GR.

3.7. Vector-tensor theories

Before we start, Will (2001) has said: "Many alternative metric theories developed during the 1970s and 1980s could be viewed as "straw-man" theories, invented to prove that such theories exist or to illustrate particular properties. Few of these could be regarded as well-motivated theories from the point of view, say, of field theory or particle physics. Examples are the vector-tensor theories studied by Will, Nordtvedt and Hellings."

Hellings and Nordtvedt (1973) and Will and Nordtvedt (1972) are both vector-tensor theories. In addition to the metric tensor there is a timelike vector field K_μ .
The gravitational action is:

$$S=\frac{1}{16\pi G}\int d^4x\sqrt{-g}[R+\omega K_\mu K^\mu R+\eta K^\mu K^\nu R_{\mu\nu}-\epsilon F_{\mu\nu}F^{\mu\nu}+\tau K_{\mu;\nu}K^{\mu;\nu}]+S_m$$

where

$$F_{\mu\nu}=K_{\nu;\mu}-K_{\mu;\nu}$$

(see Will (1981) for the field equations for $T^{\mu\nu}$ and K_μ).

Will and Nordtvedt (1972) is a special case where

$$\omega=\eta=\epsilon=0 \ ; \ \tau=1 \ .$$

Hellings and Nordtvedt (1973) is a special case where

$$\tau=0 \ ; \ \epsilon=1 \ ; \ \eta=-2\omega$$

These vector-tensor theories are semi-conservative, which means that they satisfy the laws of conservation of momentum and angular momentum but can have preferred frame effects. When $\omega=\eta=\epsilon=\tau=0$ they reduce to GR.

3.8. Other metric theories

3.9. Non-metric theories

Cartan's theory is particularly interesting both because it is a non-metric theory and because it is so old. The status of Cartan's theory is uncertain. Will (1981) claims that all non-metric theories are eliminated by Einstein's Equivalence Principle (EEP). Will (2001) tempers that by explaining experimental criteria for testing non-metric theories against EEP. Misner et al. (1973) claims that Cartan's theory is the only non-metric theory to survive all experimental tests up to that date and Turyshev (2006)* lists Cartan's theory among the few that have survived all experimental tests up to that date. The following is a quick sketch of Cartan's theory as restated by Trautman (1972).

Cartan (1922, 1923) suggested a simple generalization of Einstein's theory of gravitation. He proposed a model of space time with a metric tensor and a linear "connection" compatible with the metric but not necessarily symmetric. The torsion tensor of the connection is related to the density of intrinsic angular momentum. Independently of Cartan, similar ideas were put forward by Sciama, by Kibble and by Heyl in the years 1958 to 1966.

The original description is in terms of differential forms, but for the present article that is replaced by the more familiar language of tensors (risking loss of accuracy). As in GR, the Lagrangian is made up of a massless and a mass part. The Lagrangian for the massless part is:

$$L=\frac{1}{16\pi}\eta_{\mu}^{\nu}\Omega_{\nu}^{\mu}$$

$$\Omega_{\nu}^{\mu}=d\,\omega_{\nu}^{\mu}+\omega_{\nu}^{\xi}\omega_{\xi}^{\mu}$$

$$\nabla x^{\mu}=-\omega_{\nu}^{\mu}x^{\nu}$$

The ω_{ν}^{μ} is the linear connection. η_{μ}^{ν} is not related in any way to Minkowski but is derived from a completely antisymmetric pseudo-tensor $\eta_{\xi\mu\eta\zeta}$ as follows:

$$\eta_{\mu}^{\nu}=\frac{1}{2}g^{\nu\xi}x^{\eta}x^{\zeta}\eta_{\xi\mu\eta\zeta}$$

$$\eta_{1234}=\sqrt{-g}$$

and $g^{\nu\xi}$ is the metric tensor as usual. Indicies go from 1 to 4 as usual. By assuming that the linear connection is metric, it is possible to remove the unwanted freedom inherent in the non-metric theory. The stress-energy tensor is calculated from:

$$T^{\mu\nu}=-\frac{1}{16\pi}(g^{\mu\nu}\eta_{\eta}^{\xi}-g^{\mu\xi}\eta_{\eta}^{\nu}-g^{\xi\nu}\eta_{\eta}^{\mu})\Omega_{\xi}^{\eta}$$

On a Riemannian space-time, the Lagrangian reduces to the Lagrangian of General Relativity.

3.10. Testing of alternatives to General Relativity

[See Wikipedia, Testing of General Relativity]
The development of theories and tests has gone hand in hand. Most tests can be classified as (see Will 2001):

Basic Viability
Einstein's Equivalence Principle (EEP)
Parametric Post-Newtonian (PPN)
Strong Gravity
Gravitational Waves

3.10.1. Theories that fail Basic Viability Tests

For more details see Misner et al. (1973) Ch.39 and Will (1981) Table 2.1.

Not all theories of gravity are created equal. Very few, among the multitude in the literature, are sufficiently viable to be worth comparison with General Relativity.

In the early 1970s a group of people at Caltech including Thorne, Will and Ni (see Ni (1972)) complied a list of twentieth-century theories of gravity. Of each theory they asked the following questions: (i) is it self-consistent? (ii) is it complete? (iii) does it agree, to within several standard deviations, with all experiments performed to date? If a theory failed these criteria they did not always reject it out of hand. If the theory was incomplete in its initial statement, they sometimes tried to complete it by making minor modifications, usually by insisting that in the absence of gravity the laws of physics reverted to those of Special Relativity. For seven of the theories here, the density could be calculated from either $\rho^* = T_{\mu\nu} u^\mu u^\nu$ or $\rho^* = trace(T)$; both were considered. Even if that failed, eg. consider the case of Thirry (1948) and Jordan (1955). These are incomplete unless Jordan's parameter η is set to -1, in which case they match the theory of Brans-Dicke (1961) and so are worthy of further consideration.

For this section of the present article, the criterion of "all experiments performed to date" is replaced by "with the gross features of Newtonian Mechanics and Special Relativity". Subtler problems are discussed later.

Self-consistency among non-metric theories includes eliminating theories allowing tachyons, ghost poles, higher order poles and those that have problems with behaviour at infinity.

Among metric theories, self-consistency is best illustrated by describing several theories that fail this test. The classic example is the spin-two field theory of Fierz and Pauli (1939); the field equations imply that gravitating bodies move in straight lines whereas the equations of motion insist that gravity deflects bodies away from straight line motion. Yilmaz (1971, 1973) contains a tensor gravitational field used to construct a metric; it is mathematically inconsistent because the functional dependence of the metric on the tensor field is not well defined.

To be complete a theory of gravity must be capable of analysing the outcome of every experiment of interest. It must therefore mesh with electromagnetism and all other physics. For instance, any theory that cannot predict from first principles the movement of planets or the behaviour of atomic clocks is incomplete. Milne (1948) is incomplete because it makes no gravitational red-shift prediction.

The theories of Whitrow and Morduch (1960, 1965), Kustaanheimo (1966) and Kustaanheimo and Nuotio (1967) are either incomplete or inconsistent. The incorporation of Maxwell's equations is incomplete unless it is assumed that they are imposed on the flat background space-time and when that is done they are inconsistent because they predict zero gravitational redshift when the wave version of light (Maxwell theory) is used and nonzero redshift when the particle version (photon) is used. Another more obvious example is Newtonian gravity with Maxwell's equations; light as photons is deflected by gravitational fields (by twice that of GR) but light as waves is not.

As an example of disagreement with Newtonian experiments, Birkhoff (1943) theory predicts relativistic effects fairly reliably but demands that sound waves travel at the speed of light, which disagrees violently with experiment.

A modern example of the lack of a relativistic component is MOND by Milgrom, more on that later.

3.10.2. The Einstein Equivalence Principle (EEP)

The EEP has three components.

The first is the uniqueness of free fall, also known as the Weak Equivalence Principle (WEP). This is satisfied if inertial mass is equal to gravitational mass. η is a parameter used to test the maximum allowable violation of the WEP. The first tests of the WEP were done by Eötvös before 1900 and limited η to less than 5e-9. Modern tests have reduced that to less than 5e-13.

The second is Lorentz invariance. In the absence of gravitational effects the speed of light is constant. The test parameter for this is δ. The first tests of Lorentz invariance were done by Michelson and Morley before 1890 and limited δ to less than 5e-3. Modern tests have reduced this to less than 1e-21.

The third is local position invariance, which includes spatial and temporal invariance. The outcome of any local non-gravitational experiment is independent of where and when it is performed. Spacial local position invariance is tested using gravitational redshift measurements. The test parameter for this is α. Upper limits on this found by Pound and Rebka in 1960 limited α to less than 0.1. Modern tests have reduced this to less than 1e-4.

Theories of gravity may be metric or non-metric. In metric theories, space-time is endowed with a symmetric metric (usually written $g_{\mu\nu}$) and trajectories of freely falling bodies are geodesics of that metric.

Metric theories satisfy the Einstein Equivalence Principle. Extremely few non-metric theories, if any, satisfy this.

One theory eliminated by the EEP is the non-metric theory of Belinfante & Swihart (1957). Note, however, that Lightman & Lee (1973) obtained a metric theory of gravity that satisfies the EEP by a slight but interesting modification of the Belinfante-Swihart theory; the result is known as the BSLL bimetric theory.

3.10.3. Parametric Post-Newtonian (PPN) Formalism

See [Testing General Relativity, Misner et al. (1973) and Will (1981)] for more information.

Work on developing a standard rather than ah-hoc set of tests for evaluating alternative gravitation models began with Eddington in 1922 and resulted in a standard set of PPN numbers in Nordtvedt and Will (1972) and Will and Nordtvedt (1972). Each parameter is a measure of how much a theory departs from Newtonian gravity in a different way. Because we're talking about deviation form Newtonian theory here, this only a measure of weak-field effects. The effects of strong gravitational fields are examined later.

These ten are called :
γ, β, ξ, α_1 , α_2, α_3, ζ_1, ζ_2, ζ_3, ζ_4

γ is a measure of space curvature, being zero for Newtonian gravity and one for GR.

β is a measure of nonlinearity in the addition of gravitational fields.

ξ is a check for preferred location effects.

α_1 , α_2, α_3 measure the extent and nature of "preferred-frame effects". Any theory of gravity with at least one α nonzero is called a preferred-frame theory.

ζ_1, ζ_2, ζ_3, ζ_4, α_3 measure the extent and nature of breakdowns in global conservation laws. A theory of gravity possesses 4 conservation laws for energy-momentum and 6 for angular momentum only if all five are zero.

3.10.4. The Strong Equivalence Principle (SEP)

The SEP is an older and less complete version of PPN. It includes

The first is the generalized uniqueness of free fall. All bodies fall with the same acceleration.

The second is Generalized Local Lorentz Invariance (GLLI). All experiments are independent of the velocity of the local Lorentz frame.

The third is Generalized Local Position Invariance (GLPI). All experiments are independent of where and when they are performed.

3.10.5. Strong Gravity and Gravitational Waves

PPN is only a measure of weak field effects. Strong gravity effects can be seen in compact objects such as white dwarfs, neutron stars, and black holes. Experimental tests such as the stability of white dwarfs, spin-down rate of pulsars, orbits of binary pulsars and the existence of a black hole horizon can be used as tests of alternative to GR.

GR predicts that gravitational waves travel at the speed of light. Many alternatives to GR say that gravitational waves travel faster than light. If true, this could result in failure of causality.

3.10.6. Cosmological Tests

Many of these have been developed recently. For those theories that aim to replace dark matter, the rotation curves of galaxies, the Tully-Fisher law, the faster rotation rate of dwarf galaxies, and the gravitational lensing due to galactic clusters act as constraints.

For those theories that aim to replace inflation, the size of ripples in the spectrum of the CMB is the strictest test.

For those theories that incorporate or aim to replace dark energy, the supernova brightness results and the age of the universe can be used as tests.

Another test is the flatness of the universe. With GR, the combination of baryonic matter, dark matter and dark energy add up to make the universe exactly flat. As the accuracy of experimental tests improve, alternatives to GR that aim to replace dark matter or dark energy will have to explain why.

3.11. Results of Testing Theories

3.11.1. Theories that violate EEP

Belinfante and Swihart (1957a, 1957b)

3.11.2. Theories that violate the Strong Equivalence Principle

Theories that violate GLLI include those of Whitehead (1922) and Deser and Laurent (1968).

Theories that violate GLPI include those of Papepetrou (1954a, 1954b), Yilmaz (1958), Whitrow and Morduch (1965), Page and Tupper (1968), Ni (1972) and Rosen (1971).

3.11.3. Theories that fail to generate space curvature

Models of gravity that give $\gamma \leqslant 0$ are clearly inconsistent with experiments.

If Nordström (1913) has succeeded in making it through the problems that beset other scalar field theories then it fails here. So does Einstein and Fokker (1914). These predict that gravity causes no curvature of space and have been discussed above.

3.11.4. Theories that Fail to Explain Planetary Perihelion Advance

Ni (1973), Lee Lightman and Ni (1974)

3.11.5. Theories that Fail Strong Field tests - GW & Pulsars

Lightman and Lee (1973), Rosen (1975), Rastall (1979)

3.11.6. Lack of full Conservation

Semi-conservative theories include Hellings and Nordtvedt (1973) and Will and Nordtvedt (1972)

3.11.7. PPN parameters for a range of theories

(See Will (1981) and Ni (1972) for more details. Misner et al. (1973) gives a table for translating parameters from the notation of Ni to that of Will)

[Insert Table Here]

[Tests agree with GR ... eliminates ... special cases reduce to GR ...]

A full list of PPN parameters is not available for Whitehead (1922), Deser-Laurent (1968), Bollini-Giamiago-Tiomino (1970), but in these three cases $\beta=\xi$, which is in strong conflict with GR and experimental results.

4. Modern Theories 1980s to Present

This section includes alternatives to GR published after the observations of galaxy rotation that led to the hypothesis of "dark matter".

There is no known reliable list of comparison of these theories.

Those considered here include:
Beckenstein (2004), Moffat (1995), Moffat (2002), Moffat (2005a, b).

These theories are presented with a cosmological constant or added scalar or vector potential.

4.1. Motivations

Motivations for the more recent alternatives to GR are almost all cosmological, associated with or replacing such constructs as "inflation", "dark matter" and "dark energy". The basic idea is that gravity agrees with GR at the present epoch but may have been quite different in the early universe.

There was a slow dawning realisation in the physics world that there were several problems inherent in the then big bang scenario, two of these were the horizon problem and the observation that at early times when quarks were first forming there was not enough space on the universe to contain even one quark. Inflation theory was developed to overcome these. Another alternative was constructing an alternative to GR in which the speed of light was larger in the early universe.

The discovery of unexpected rotation curves for galaxies in took everyone by surprise. Could there be more mass in the universe than we are aware of, or is the theory of gravity itself wrong? The consensus of opinion now is that the missing mass is "cold dark matter", but that consensus was only reached after trying alternatives to general relativity and some physicists still believe that alternative models of gravity might hold the answer.

The discovery of the accelerated expansion of the universe by Perlmutter led to the rapid reinstatement of Einstein's cosmological constant, and quintessence arrived as an alternative to the cosmological constant. At least one new alternative to GR attempted to explain Rerlmutter's results in a completely different way.

Another observation that sparked recent interest in alternatives to General Relativity is the Pioneer anomaly. It was quickly discovered that alternatives to GR could explain the qualitative features of the anomaly but not the magnitude. Any alternative model of gravity that could explain the Pioneer anomaly would have to depart so strongly from GR that it would fail to satisfy other experimental observations.

4.2. Cosmological Constant and Quintessence

The cosmological constant Λ is a very old idea, going back to Einstein in 1917. The success of the Friedmann model of the universe in which $\Lambda=0$ led to the general acceptance that it is zero, but the use of a non-zero value came back with a vengeance when Perlmutter discovered that the expansion of the universe is accelerating

First, let's see how it influences the equations of Newtonian gravity and General Relativity.

In Newtonian gravity, the addition of the cosmological constant changes the Newton-Poisson equation from:

$$\nabla^2 \phi=4\pi\rho$$

to

$$\nabla^2 \phi-\Lambda\phi=4\pi\rho$$

In GR, it changes the Einstein-Hilbert action from

$$S=\frac{1}{16\pi G}\int R\sqrt{-g}\,d^4x+S_m$$

to

$$S=\frac{1}{16\pi G}\int (R-2\Lambda)\sqrt{-g}\,d^4x+S_m$$

which changes the field equation

$$T^{\mu\nu}=\frac{1}{8\pi G}(R^{\mu\nu}-g^{\mu\nu}R/2)$$

to

$$T^{\mu\nu}=\frac{1}{8\pi G}(R^{\mu\nu}-g^{\mu\nu}R/2+\Lambda g^{\mu\nu})$$

In alternative theories of gravity, a cosmological constant can be added to the action in exactly the same way.

That's not the only way to get an accelerated expansion of the universe in alternatives to GR. We've already seen how the scalar potential $\lambda(\phi)$ can be added to scalar tensor theories. This can also be done in every alternative the GR that contains a scalar field φ by adding the term $\phi\lambda(\phi)$ inside the Lagrangian for the gravitational part of the action (the L_ϕ part of

$$S = \frac{1}{16\pi} \int d^4 x \sqrt{-g} L_\phi + S_m$$).

Because $\lambda(\phi)$ is an arbitrary function of the scalar field, it can be set to give an acceleration that is large in the early universe and small at the present epoch. This is known as quintessence.

A similar method can be used in alternatives to GR that use vector fields, including vector-tensor theories and Rastall (1979). A term proportional to

$$K^\mu K^\nu g_{\mu\nu}$$

is added to the Lagrangian for the gravitational part of the action.

4.3. Relativistic MOND

(see Beckenstein (2004) for more details).

The original theory of MOND by Milgrom was developed in 1983 as an alternative to "dark matter". Departures from Newton's law of gravitation are governed by an acceleration scale a_0 , not a distance scale. MOND successfully explains the Tully-Fisher observation that the luminosity of a galaxy should scale as the fourth power of the rotation speed. It also explains why the rotation discrepancy in dwarf galaxies is particularly large.

There were several problems with MOND in the beginning.
i. It did not include relativistic effects
ii. It violated the conservation of energy, momentum and angular momentum
iii. It was inconsistent in that it gives different galactic orbits for gas and for stars
iv. It did not state how to calculate gravitational lensing from galaxy clusters.

By 1984, problems ii. and iii. had been solved by introducing a Lagrangian (AQUAL). A relativistic version of this based on scalar-tensor theory was rejected because it allowed waves in the scalar field to propagate faster than light. The Lagrangian of the non-relativistic form is:

$$L = -\frac{a_0^2}{8\pi G} f\left[\frac{|\nabla\phi|^2}{a_0^2}\right] - \rho\phi$$

The relativistic version of this has:

$$L = -\frac{a_0^2}{8\pi G} \tilde{f}(L^2 g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi)$$

with a nonstandard mass action. Here *f* and $\tilde{f}$ are arbitrary functions selected to give Newtonian and MOND behaviour in the correct limits.

By 1988, a second scalar field (PCC) fixed problems with the earlier scalar-tensor version but is in conflict with the perihelion precession of Mercury and gravitational lensing by galaxies and clusters.

By 1997, MOND had been successfully incorporated in a stratified relativistic theory [Sanders], but as this is a preferred frame theory it has problems of its own.

Beckenstein (2004) introduced a tensor-vector-scalar model (TeVeS). This has two scalar fields φ and σ and vector field U_α . The action is split into parts for gravity, scalars, vector and mass.

$$S = S_g + S_s + S_v + S_m$$

The gravity part is the same as in GR.

$$S_s = -\frac{1}{2} \int \left[\sigma^2 h^{\alpha\beta} \phi_{,\alpha} \phi_{,\beta} + \frac{1}{2} G l^{-2} \sigma^4 F(kG\sigma^2)\right] (-g)^{1/2} d^4 x$$

$$S_v = -\frac{K}{32\pi G}\int [g^{\alpha\beta} g^{\mu\nu} U_{[\alpha,\mu]} U_{[\beta,\nu]} - 2(\lambda/K)(g^{\mu\nu} U_\mu U_\nu + 1)](-g)^{1/2} d^4x$$
$$S_m = \int L(\tilde{g}_{\mu\nu}, f^\alpha, f^\alpha_{|\mu}, \cdots)(-g)^{1/2} d^4x$$
where
$h^{\alpha\beta} \equiv h^{\alpha\beta} - U^\alpha U^\beta$, l is a length scale, F is an arbitrary function, k and K are constants, square brackets in indices represent anti-symmetrization λ is a Lagrange multiplier (calculated elsewhere), $\tilde{g}^{\alpha\beta} = e^{2\phi} g^{\alpha\beta} + 2U^\alpha U^\beta \sinh(2\phi)$, and L is a Lagrangian translated from flat spacetime onto the metric $\tilde{g}^{\alpha\beta}$

4.4. Moffat's Theories

Moffat (1995) developed a non-symmetric gravitation theory (NGT). This is not a metric theory. It is claimed that it does not contain a black hole horizon. Burko and Ori have found that NGT can contain black holes. Later, Moffat claimed that it has also been applied to explain rotation curves of galaxies without invoking "dark matter". Damour, Deser & MaCarthy (1993) have criticised NGT, saying that it has unacceptable asymptotic behaviour.

The mathematics is not difficult but is intertwined so the following is only a brief sketch. Starting with a non-symmetric tensor $g_{\mu\nu}$, the Lagrangian density is split into
$$L = L_R + L_M$$
where L_M is the same as for matter in GR.
$$L_R = \sqrt{-g}[R(W) - 2\lambda - (1/4)\mu^2 g^{\mu\nu} g_{[\nu\mu]}] - (1/6) g^{\mu\nu} W_\mu W_\nu$$
where $R(W)$ is a curvature term analogous to but not equal to the Ricci curvature in GR, λ and μ^2 are cosmological constants, $g_{[\nu\mu]}$ is the antisymmetric part of $g_{\nu\mu}$.
W_μ is a connection, and is a bit difficult to explain because it's defined recursively. However, $W_\mu \approx -2g_{[\mu\nu]}^{,\nu}$.

Moffat (2002) is a scalar-tensor bimetric gravity theory (BGT) and is one of the many theories of gravity in which the speed of light is faster in the early universe. These theories were motivated partly be the desire to avoid the "horizon problem" without invoking inflation. It has a variable G. The theory also attempts to explain the dimming of supernovae from a perspective other than the acceleration of the universe and so runs the risk of predicting an age for the universe that is too small.

Overall, this theory looks totally balmy. The action is split into gravity, scalar field, and matter parts. The gravity and scalar field equations are bog standard for a Brans-Dicke theory with cosmological constant and scalar potential, but applied with a Minkowski metric! Only the matter term uses a non-flat metric, and that is $g_{\mu\nu} = \eta_{\mu\nu} + B\partial_\mu\phi\partial_\nu\phi$ where B has dimensions of length squared. This theory is going to fail Lorentz invariance and deflection of light tests at the very least.

Moffat (2005a) metric-skew-tensor-gravity (MSTG) theory is able to predict rotation curves for galaxies without either dark matter or MOND, and claims that it can also explain gravitational lensing of galaxy clusters without dark matter. It has variable G, increasing to a final constant value about a million years after the big bang.

The theory seems to contain an asymmetric tensor $A_{\mu\nu}$ field and a source current J_μ vector. The action is split into:
$$S = S_G + S_F + S_{FM} + S_M$$

Both the gravity and mass terms match those of GR with cosmological constant. The skew field action and the skew field matter coupling are:

$$S_F=\int d^4x\sqrt{-g}(\frac{1}{12}F_{\mu\nu\rho}F^{\mu\nu\rho}-\frac{1}{4}\mu^2 A_{\mu\nu}A^{\mu\nu})$$

$$S_{FM}=\int d^4x\,\epsilon^{\alpha\beta\mu\nu}A_{\alpha\beta}\partial_\mu J_\nu$$

where

$$F_{\mu\nu\rho}=\partial_\mu A_{\nu\rho}+\partial_\nu A_{\rho\mu}+\partial_\rho A_{\mu\nu}$$

and $\epsilon^{\alpha\beta\mu\nu}$ is the Levi-Civita symbol. The skew field coupling is a Pauli coupling and is gauge invariant for for any source current. The source current looks like a matter fermion field associated with baryon and lepton number.

Moffat (2005b) scalar-tensor-vector-gravity (SVTG) theory.

The theory contains a tensor, vector and three scalar fields. But the equations are quite straightforward. The action is split into:

$S=S_G+S_K+S_S+S_M$ with terms for gravity, vector field K_μ , scalar fields G, ω & μ, and mass.

S_G is the standard gravity term with the exception that *G* is moved inside the integral.

$$S_K=-\int d^4x\sqrt{-g}\,\omega(\frac{1}{4}B_{\mu\nu}B^{\mu\nu}+V(K))$$

where $B_{\mu\nu}=\partial_\mu K_\nu-\partial_\nu K_\mu$

$$S_S=-\int d^4x\sqrt{-g}\left[\frac{1}{G^3}(\frac{1}{2}g^{\mu\nu}\nabla_\mu G\nabla_\nu G-V(G))+\frac{1}{G}(\frac{1}{2}g^{\mu\nu}\nabla_\mu\omega\nabla_\nu\omega-V(\omega))+\frac{1}{\mu^2 G}(\frac{1}{2}g^{\mu\nu}\nabla_\mu\mu\nabla_\nu\mu-V(\right.$$

The potential function for the vector field is chosen to be:

$$V(K)=-\frac{1}{2}\mu^2\phi^\mu\phi_\mu-\frac{1}{4}g(\phi^\mu\phi_\mu)^2$$

where *g* is a coupling constant.

4.5. Other Modern Theories

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